Filtering out the cosmological constant in the Palatini formalism of modified gravity

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Abstract

According to theoretical physics the cosmological constant (CC) is expected to be much larger in magnitude than other energy densities in the universe, which is in stark contrast to the observed Big Bang evolution. We address this old CC problem not by introducing an extremely fine-tuned counterterm, but in the context of modified gravity in the Palatini formalism. In our model the large CC term is filtered out, and it does not prevent a standard cosmological evolution. We discuss the filter effect in the epochs of radiation and matter domination as well as in the asymptotic de Sitter future. The final expansion rate can be much lower than inferred from the large CC without using a fine-tuned counterterm. Finally, we show that the CC filter works also in the Kottler (Schwarzschild-de Sitter) metric describing a black hole environment with a CC compatible to the future de Sitter cosmos.

1 Introduction

The starting point of this work is a CC or equivalently a vacuum energy density Λ of enormous magnitude. This expectation is suggested by contributions to the CC coming from phase transitions in the early universe, zero-point energy in quantum field theory or even from quantum gravity. In general, all these parts are of different magnitude and probably unrelated to each other. Hence, the sum Λ of all terms is dominated by the largest contribution. Since other energy sources dilute with the expansion of the universe, the CC will eventually take control over the cosmos. Depending on its sign the CC would induce in the very early universe either a Big Crunch or an eternal de Sitter phase with a very high Hubble rate $H \propto \Lambda$. Obviously, the standard Big Bang evolution does not happen in this case.

The simplest way to avoid this problem is the introduction of a CC counterterm Λ_{ct}, which makes the sum $|\Lambda + \Lambda_{\text{ct}}|$ smaller than the currently observed critical energy density $\rho_{c0} \sim 10^{-47}\,\text{GeV}^4$. For concreteness let us assume that $\Lambda \sim -M_{\text{ew}}^4$ were related to the electroweak phase transition at the energy scale $M_{\text{ew}} \sim 10^2\,\text{GeV}$. Then the counterterm must be extraordinarily close to $(-\Lambda)$ requiring an enormous amount of fine-tuning,

$$\left|1 + \frac{\Lambda_{ct}}{\Lambda}\right| < \left|\frac{\rho_{c0}}{\Lambda}\right| \sim 10^{-55}. \tag{1}$$

Apart from the fine-tuning of the classical counterterm, the situation is even more involved when quantum corrections are included, cf. Ref. [1] for an elaborated discussion in the

context of the electroweak sector of the standard model of particles. Moreover, the problem worsens when Λ is dominated by higher energy scales, possibly originating from grand unified theories where $\Lambda \sim (10^{16}\,\mathrm{GeV})^4$ or quantum gravity with $\Lambda \sim (10^{19}\,\mathrm{GeV})^4$ for instance. Summing up, the fine-tuning of the CC is considered to be one of the most severe problems in theoretical physics [2, 3]. In addition, the current accelerated expansion of the universe [4, 5, 6] can be explained very well by a tiny CC of the same magnitude as the energy density of matter, giving rise to the so-called coincidence problem. For the latter problem many explanations have been proposed [7, 8, 9, 10, 11], which induce late-time accelerated expansion. However, most of these models tacitly assume that the large Λ has been fine-tuned away and thus they do not address the big CC problem.

Without fine-tuning we have to accept the existence of the presumed huge CC, and we have to find a way to neutralise its effects in order to obtain a reasonable cosmological evolution. Along this line, several proposals have been made, e.g. relaxation models for a large CC in the context of matter with an inhomogeneous equation of state (EOS) [12], or in the LXCDM framework [13] with a variable cosmological term [14, 15], see also [16, 17, 18, 19, 20, 21]. Removing or filtering out vacuum energy has been investigated e.g. in Refs. [22, 23, 24, 25], and it is a feature in unimodular gravity [26, 27, 28, 29, 30]. Recently, a CC relaxation model has been discussed in the context of modified gravity with an action functional $f(R, G)$ involving the Ricci scalar R and the Gauß-Bonnet invariant G in the metric formalism [31, 1], where the action is varied with respect to the metric g_{ab} only.

In this work, we also consider a modified gravity model with an action functional f in terms of the Ricci scalar R and the squared Ricci tensor $Q = R_{ab}R^{ab}$. However, here we apply the Palatini formalism, where the metric g_{ab} and the connection Γ^a_{bc} are treated independently by the variation principle. In contrast, the metric formalism requires from the beginning that

$$\Gamma^a_{bc}[g] = \frac{1}{2} g^{ad}(g_{dc,b} + g_{bd,c} - g_{bc,d}) \tag{2}$$

is the Levi-Civita connection of g_{ab}, whereas the Palatini connection depends also on the matter sector. As we will show in this paper, this allows the construction of a filter for a large CC, and the results will be similar in effect to unimodular gravity. However, in our setup the CC is not eliminated completely, but it appears in suppressed corrections. Furthermore, we investigate the filter effect from the early universe till the asymptotic future in addition to black hole environments. It turns out that finite vacuum energy shifts originating e.g. from phase transitions can be neutralised, too.

There is an interesting conceptual difference to the CC relaxation models in the metric formalism, where the large vacuum energy is not filtered out from the total energy content, but gravity is modified such that Λ does not induce large curvatures. This happens on the level of the Einstein equations, i.e. by solving differential equations to obtain concrete low-curvature solutions. In the Palatini framework of this paper, we will show that the large CC can be removed already in an algebraic way before solving differential equations. Moreover, the latter are only of second order, whereas the metric version of modified gravity generally involves a higher differential order, signalling the existence of new degrees of freedom, which can be the source of new instabilities and other problems. More differences between both formalisms will become visible in the forthcoming discussions in this paper.

Some work on the Palatini formalism can be found e.g. in Refs. [32, 33, 34, 35, 36, 37], and comparisons with the metric and other formalisms were made in Refs. [38, 39, 40, 41, 42, 43, 44]. Non-trivial properties in Palatini models have been investigated in Refs. [45, 46, 47, 48], and finally the resolution of the Big Bang singularity has been proposed in this

context [49, 50], possibly in connection with loop quantum gravity [51].

The paper is organised as follows, in Sec. 2 we briefly reproduce how to solve $f(R, Q)$ modified gravity models. In Sec. 3 we present our model which filters out the large CC. The results will be applied to cosmology in Sec. 4 discussing the radiation, matter and late-time de Sitter era. Finally, in Sec. 5 we show that the CC filter works also for the Kottler (Schwarzschild-de Sitter) solution describing a black hole in the presence of a CC. We conclude in Sec. 6 and give an outlook to future developments.

In this work the speed of light c and the Planck constant $\hbar$ are set to unity, the signature of the metric is $(-1, +1, +1, +1)$.

2 $f(R, Q)$ modified gravity in the Palatini formalism

The action of our setup is given by

$$\mathcal{S} = \int d^4x \left[\sqrt{|g|}\frac{1}{2}f(R, Q)\right] + \mathcal{S}_{\text{mat}}[g_{ab}, \phi], \tag{3}$$

where g_{ab} is the "physical" metric, on which the matter fields ϕ in $\mathcal{S}_{\text{mat}}$ propagate. The Ricci scalar R and the squared Ricci tensor Q depend on both the metric and the connection Γ^a_{bc} while the Ricci tensor R_{ab} is defined only in terms of the latter:

$$\begin{aligned} R_{ab}[\Gamma] &= \Gamma^e_{ab,e} - \Gamma^e_{eb,a} + \Gamma^e_{ab}\Gamma^f_{fe} - \Gamma^e_{af}\Gamma^f_{eb} && (4)\\ R[g, \Gamma] &= R^a{}_a = g^{ab}R_{ab} && (5)\\ Q[g, \Gamma] &= R^{ab}R_{ab} = g^{ac}g^{bd}R_{ab}R_{cd}. && (6)\end{aligned}$$

Note that in general these quantities are different from their metric versions, and the absence of torsion implies that the connection is symmetric. Moreover, we restrict our discussion to the case of a symmetric Ricci tensor R_{ab} in the action (3). This property is not automatic even for symmetric connections, which was shown recently in Ref. [52].

With these preliminaries the variation $2\,\delta\mathcal{S}/\delta g^{ab} = 0$ of the action functional $\mathcal{S}$ with respect to g_{ab} yields the modified Einstein equations

$$f_R R_m{}^n + 2f_Q R_m{}^a R_a{}^n - \frac{1}{2}\delta_m{}^n f = T_m{}^n, \tag{7}$$

where the energy-momentum tensor $T_m{}^n$ emerges from the term $\mathcal{S}_{\text{mat}}$, and f_R and f_Q are partial derivatives of f with respect to the scalars R and Q. Moreover, we obtain from $\delta\mathcal{S}/\delta\Gamma^a_{bc} = 0$ the equation of motion (EOM) for the Palatini connection,

$$\nabla_a \left[\sqrt{|g|}(f_R g^{mn} + 2f_Q R^{mn})\right] = 0, \tag{8}$$

where ∇_a denotes the covariant derivative in terms of the yet unknown connection Γ^a_{bc}.

At this point we should remark that in the metric formalism the term Q yields EOMs with higher-order derivatives and problematic extra degrees of freedom. Generally, it is difficult to avoid instabilities, e.g. of the Ostrogradski-type[1] [53]. In contrast, the Palatini

[1]In the metric approach this type of instability can be avoided in gravity actions depending only on the Ricci scalar R and the Gauß-Bonnet term G. It would be interesting to compare both approaches in the context of $f(R, G)$ models. However, G contains the squared Riemann tensor, and to our knowledge no method to solve the corresponding Palatini EOMs has been found yet.

formalism provides second-order EOMs for our scenario just as standard general relativity, and problems from extra degrees of freedom do not occur.

In the following, we work along the lines of Ref. [49], where a procedure for solving the Palatini EOMs is discussed. The strategy is as follows: the formal solutions to both EOMs in (7) and (8) relate in an algebraic way the geometrical scalars R and Q with the physical metric g_{ab} and the energy-momentum tensor $T_m{}^n$, respectively. Thus, one can express R and Q in terms of the matter content alone by solving this set of equations. Subsequently, the results are plugged back into the formal solutions to obtain the explicit form of the Palatini connection Γ and all the quantities derived from it. Since the results will involve the metric g_{ab} and its derivatives, it is possible to relate the cosmic expansion rate with the matter energy density, for instance. In the rest of this section, we explain the procedure for general $f(R,Q)$ models.

First, let us write Eq. (7) in matrix form by introducing the 4×4-matrices $\hat{P} = R_m^n = R_{ma}g^{an}$ and $\hat{T} = T_m{}^n$, whose entries are just the components of the corresponding tensors components. With the identity matrix $\hat{I}$ we obtain

$$f_R\hat{P} + 2f_Q(\hat{P})^2 - \frac{1}{2}f\hat{I} = \hat{T}, \tag{9}$$

and the trace of this equation reads

$$f_R R + 2Qf_Q - 2f = T, \tag{10}$$

where $R = \mathrm{tr}(\hat{P})$, $Q = \mathrm{tr}(\hat{P}^2)$ and $T = T_m{}^m$.

For determining the Palatini connection Γ we introduce the auxiliary metric h_{mn} and consider the equation

$$\nabla_a[\Gamma]\left[\sqrt{|h|}h^{mn}\right] = 0, \tag{11}$$

where $\nabla_a[\Gamma]$ is the covariant derivative in terms of Γ. Consequently, in order to solve this equation the connection Γ must be compatible with h_{mn}, i.e. it has to be the Levi-Civita connection of h_{mn},

$$\Gamma^a_{bc}[h] = \frac{1}{2}h^{ad}(h_{dc,b} + h_{bd,c} - h_{bc,d}), \tag{12}$$

just as in general relativity. Hence, we convert Eq. (8) into (11) by defining h^{mn} in the following way,

$$\sqrt{|h|}\hat{h}^{-1} = \sqrt{|g|}\hat{g}^{-1}\hat{\Sigma} \quad \text{with} \quad \hat{\Sigma} = \left(f_R\hat{I} + 2f_Q\hat{P}\right), \tag{13}$$

where the metric h_{mn} (and analogously g_{mn}) has been written in matrix notation as $\hat{h} = h_{mn}$ with its inverse $\hat{h}^{-1} = h^{mn}$. Calculating the determinant of both sides, we find $h^2h^{-1} = h = g\det\hat{\Sigma}$, which allows to eliminate $\sqrt{|h|}$ and finally yields

$$h^{mn} = \hat{h}^{-1} = \frac{\hat{g}^{-1}\hat{\Sigma}}{\sqrt{|\det\hat{\Sigma}|}}, \quad h_{mn} = \hat{h} = \sqrt{|\det\hat{\Sigma}|}\,\hat{\Sigma}^{-1}\hat{g}. \tag{14}$$

Since the connection in (12) solves Eq. (11), the formal solution of Eq. (8) is also given by $\Gamma^a_{bc}[h]$ in (12) if h_{mn} is defined as in (14).

The remaining step is to find $\hat{P}$ which requires an explicit form for the energy-momentum tensor $T_m{}^n$. Here, we assume that the matter sector can be described by a perfect fluid,

$$T_m{}^n = (\rho + p)u_m u^n + (p - \Lambda)\delta_m^n, \tag{15}$$

where ρ and p are the energy density and pressure of (ordinary) matter, including e.g. dust and incoherent radiation. And u_m denotes the corresponding 4-velocity vector of the matter field. Λ represents the energy density corresponding to the cosmological constant, and it contains all vacuum energy contributions. Thus we require $p \neq -\rho$ without loss of generality. Next, let us write the matrix expression (9) in the following way

$$
\begin{aligned}
(2f_Q)^2 \hat{M}^2 &= x^2 \hat{I} + \mu \widehat{u_m u^n} && (16)\\
\hat{M} &:= \hat{P} + \frac{1}{4}\frac{f_R}{f_Q}\hat{I} && (17)\\
x^2 &:= 2f_Q(p-\Lambda) + f_Q f + \frac{1}{4}f_R^2 && (18)\\
\mu &:= 2f_Q(\rho+p). && (19)
\end{aligned}
$$

By explicit calculation one can check that

$$
c_a \cdot 2f_Q \hat{M} = x\,\hat{I} + y\,\widehat{u_m u^n} \tag{20}
$$

with

$$
y := \frac{-x + c_b \cdot \sqrt{x^2 + \mu(u_m u^m)}}{(u_m u^m)} \tag{21}
$$

is a solution to Eq. (16), which yields $\hat{P}$. The constants $c_{a,b} = \pm 1$ and the sign convention[2] for $x = \sqrt{x^2}$ will be fixed later by consistency considerations.

Now, we have to determine the scalars R and Q, which follow from the trace equation (10) and the trace of (20),

$$
c_a(2f_Q R + 2f_R) = 4x - y, \tag{22}
$$

where $u_m u^m = -1$ will be used from here on. In the last equation we eliminate all roots by squaring twice, which results to

$$
\left((2f_Q R + 2f_R)^2 + 8x^2 + \mu\right)^2 = 36x^2\,(2f_Q R + 2f_R)^2\,, \tag{23}
$$

where $c_{a,b} = \pm 1$ and odd powers of x have dropped out. Solving this algebraic equation together with (10) is the tough part of the Palatini formalism. Once this task is achieved, R, Q and $\hat{P} = R_m^{\ n}$ in (20) are given as functions of ρ, p, Λ only, and can be used to calculate the connection Γ. For this purpose, let us write the matrix $\hat{\Sigma}$ in (13) as

$$
\begin{aligned}
\hat{\Sigma} &= 2f_Q\hat{M} + \frac{1}{2}f_R\hat{I} = L_1\,\hat{I} + L_2\,\widehat{u_m u^n} = L_1\left(\hat{I} + \frac{L_2}{L_1}\,\widehat{u_m u^n}\right), && (24)\\
&\text{with } L_1 := c_a x + \frac{1}{2}f_R, \quad L_2 := c_a y. && (25)
\end{aligned}
$$

Using $\det(\hat{I} + \widehat{a_m b^n}) = 1 + b^m a_m$ we find $\det(\hat{\Sigma}) = L_1^3(L_1 - L_2)$, and the inverse matrix reads

$$
\hat{\Sigma}^{-1} = \frac{1}{L_1}\,\hat{I} - \frac{L_2}{L_1(L_1 - L_2)}\,\widehat{u_m u^n}.
$$

[2] Changing the sign of x in Eqs. (20) and (21) by $x \to -x$ corresponds to $c_a \to -c_a$, and therefore it does not yield a new solution. Here, we use the convention $\sqrt{x^2 + \mu(u_m u^m)} = x\sqrt{1 + \mu(u_m u^m)/x^2}$, and the second possibility $-x\sqrt{1 + \cdots}$ would just mean $c_b \to -c_b$.

Finally, we have all ingredients to write down the explicit form of the auxiliary metric[3] from Eq. (14)

$$h_{mn} \;=\; \Omega\left(g_{mn} - \frac{L_2}{L_1 - L_2} u_m u_n\right) \tag{26}$$

$$h^{mn} \;=\; \Omega^{-1}\left(g^{mn} + \frac{L_2}{L_1} u^m u^n\right), \tag{27}$$

where

$$\Omega := \frac{\sqrt{|\mathrm{det}\hat{\Sigma}|}}{L_1} = \frac{\sqrt{|L_1^3(L_1 - L_2)|}}{L_1}. \tag{28}$$

Once h_{mn} is known, the connection Γ^a_{bc} follows directly from Eq. (12), and subsequently the Ricci tensor (4) and the scalars in Eqs. (5) and (6) can be calculated.

3 Relaxing the CC with a filter

In this section we study a modified gravity model to relax the CC in the Palatini $f(R, Q)$ framework. Motivated by earlier work in the metric formalism [31, 1] we consider the following ansatz for the gravity action

$$f(R, Q) = \kappa R + z \;\text{ with }\; z := \beta\,\frac{R^n}{B^m}, \;\; B = R^2 - Q, \tag{29}$$

where κ and β are non-zero constant parameters and n and m positive numbers. In the metric formalism model [31] the structure of the function B was enforcing the universe to expand like a matter dominated cosmos even when the matter energy density ρ_m was much smaller in magnitude than the vacuum energy density Λ. However, in the Palatini framework the function B is not known in the beginning, and it is necessary to investigate under which circumstances a relaxed cosmological expansion behaviour can be obtained. Moreover, we will see in the following that z is not a correction to the Einstein-Hilbert term but a crucial part of the action functional f. Therefore, one should refrain from considering the limit $z \to 0$.

From Eq. (29) we find

$$f_R = \frac{\kappa R + nz}{R} - 2 f_Q R, \quad f_Q = m\frac{z}{B}, \tag{30}$$

and the trace of the stress tensor (15) reads

$$T = -4\Lambda + 3p - \rho. \tag{31}$$

As a result, Eq. (10) provides the first equation for finding R and B (or Q),

$$\gamma z = \kappa R - 4\Lambda + 3p - \rho, \quad \gamma := (n - 2 - 2m), \tag{32}$$

while the second one is given in Eq. (23), explicitly

$$0 \;=\; f_Q^3 R^2\, S_3 + f_Q^2\, S_2 + f_Q R^{-2}\, S_1 \tag{33}$$

[3]The relation in Eq. (26) is called a disformal transformation [54], which is used e.g. in MOND theories [55] and scalar field models for dark energy [56].

$$
\begin{aligned}
S_3 &:= 72\left[\kappa R+\rho+p+\frac{1}{3}z(2+2n+\gamma)\right] &(34)\\
S_2 &:= 4\left[z^2\left(-17n^2+4n(2+\gamma)+4(2+\gamma)^2\right)\right.\\
&+ 12z(\rho+p)(2-n+\gamma)+6z\kappa R(4-5n+2\gamma)\\
&+ \left. 9(\rho+p)^2-9(\kappa R)^2\right] &(35)\\
S_1 &:= 24\left[z(\kappa R+nz)^2(n-2-\gamma)\right]. &(36)
\end{aligned}
$$

Apparently, this set of equations is quite complicated, thus we will solve it approximately. First, we consider in this work only the epochs when the large CC Λ dominates over all other energy sources. Then Eq. (32) implies the relation $z=\beta R^n/B^m=\mathcal{O}(\Lambda)$ suggesting that B/R^2 and $f_Q^{-1}=B/(mz)$ are relatively small quantities, which may be used as expansion parameters in Eq. (33). At this point we cannot prove this suggestion quantitatively because R and B are not known yet. However, it will be confirmed later by Eqs. (43) and (45). As a result of assuming that f_Q^{-1} is sufficiently small, the term S_3 proportional to f_Q^3 in (33) will be the most important one, and a good zero-order solution can be found by neglecting the other terms $f_Q^2\,S_2+f_QR^{-2}\,S_1$ and solving only $S_3=0$.

Our goal is a relaxed universe, i.e. one which is not dominated by the large CC term, and it can be realised by requiring that the Ricci scalar R following from $S_3=0$ is free from large $\mathcal{O}(z)$ contributions. This happens when the parameter n is restricted by $2+2n+\gamma=3n-2m=0$, which eliminates the $\mathcal{O}(z)$ term in Eq. (34). We will apply this condition from now on, hence Eq. (33) can be written as

$$
\begin{aligned}
0 &= \kappa R+r &(37)\\
&- \frac{2}{9}mz\left(\frac{B}{R^2}\right)\left[1+\frac{3}{mz}\left(3\kappa R+2r\right)+\frac{9}{4(mz)^2}\left((\kappa R)^2-r^2\right)\right]\\
&+ \frac{8}{27}mz\left(\frac{B}{R^2}\right)^2\left(1+\frac{3\kappa R}{2(mz)}\right)^2 \quad \text{with}\quad r:=\rho+p.
\end{aligned}
$$

From the first line in the last equation one clearly observes that CC terms with EOS $p=-\rho$ do not contribute to the Ricci scalar at leading order. In other words, the CC is filtered out from $r=\rho+p$, which describes the matter sector. Note that $z=\mathcal{O}(\Lambda)$ still appears in suppressed correction terms.

In the following, we will solve Eq. (37) in situations relevant for cosmology. A first order correction to $S_3=0$ can be found by keeping only the term $z(B/R^2)$ from the second line in (37), which leads to

$$
\kappa R+r=\frac{2}{9}mz\left(\frac{B}{R^2}\right)=\frac{2}{9}mz\left(\frac{\beta}{z}\right)^{\frac{1}{m}}R^{-\frac{4}{3}}, \tag{38}
$$

where we used $z=\beta R^n/B^m$ from Eq. (29) with $3n=2m$ in the last step. Thus, we obtain a 7^{th}-order polynomial equation in R,

$$
\left(\kappa R+r\right)^3\left(\kappa R\right)^4=L_3:=\kappa^4\left(\frac{2}{9}mz\left(\frac{\beta}{z}\right)^{\frac{1}{m}}\right)^3, \tag{39}
$$

which clearly shows that R is a function of ρ and p only. Finally, we find the approximate solution

$$
\kappa R=-r+D+\mathcal{O}\left(\frac{D^2}{r}\right)\quad\text{with}\quad D:=\left(\frac{L_3}{r^4}\right)^{\frac{1}{3}}, \tag{40}
$$

when the matter-related quantity r lies in the range $|D| \ll r \ll |\Lambda|$, which we refer to as the early-time limit or the limit of large energy density in r. Moreover, $\kappa R < 0$ because $r > 0$ for ordinary matter, and κR will remain negative for decreasing ρ because $\kappa R \to 0$ is not a solution of Eq. (39).

In the opposite limit, $r \to 0$, which is called the late-time limit in the following, we obtain from Eq. (39)

$$\kappa R = \rho_e - \frac{3}{7} r + \mathcal{O}\left(\frac{r^2}{\rho_e}\right) \quad \text{with} \quad \rho_e := L_3^{\frac{1}{7}}, \tag{41}$$

indicating that κR approaches the negative constant ρ_e for vanishing matter. For a given value of Λ or z the parameter β must be chosen adequately for $L_3 = \rho_e^7 < 0$. In Sec. 4.2 we will show that $(-\rho_e)$ is close to the critical energy density in the asymptotic future, which corresponds to the tiny observed value of the effective CC in Eq. (83). The parameter dependence of the solution set of the full equation (37) might further constrain β, however this has to be determined numerically. In Fig. 1 we show an example for $m = 3$ and $\rho_e < 0$, which nicely demonstrates the validity of our approximations. Note that not all values of m might allow physically reasonable solutions. We will discuss some examples for β at the end of Sec. 4.2. However, in this work we concentrate on analytical results, and the complete parameter dependence as well as the case $L_3 > 0$ will be investigated elsewhere.

Via $z = \beta(R^{2/3}/B)^m$ from Eq. (29) it is straightforward to obtain B and B/R^2 from the approximate solution of R. In the following we denote subdominant corrections by $\varepsilon \ll 1$. Accordingly, at early times Eq. (40) yields

$$\begin{aligned} B &= \sqrt{\frac{L_3}{D}} \left(\frac{2}{9} m z\right)^{-1} (-\kappa)^{-2} \left(1 - \frac{2}{3}\frac{D}{r} + \mathcal{O}(\varepsilon^2)\right), && (42) \\ \frac{B}{R^2} &= \frac{D}{\left(\frac{2}{9} m z\right)} \left(1 - \frac{D}{r} + \mathcal{O}(\varepsilon^2)\right), && (43) \end{aligned}$$

whereas from Eq. (41) we obtain the corresponding late-time results

$$\begin{aligned} B &= \rho_e^3 \left(\frac{2}{9} m z\right)^{-1} (-\kappa)^{-2} \left(1 - \frac{2}{7}\frac{r}{\rho_e} + \mathcal{O}(\varepsilon^2)\right), && (44) \\ \frac{B}{R^2} &= \rho_e \left(\frac{2}{9} m z\right)^{-1} \left(1 - \frac{4}{7}\frac{r}{\rho_e} + \mathcal{O}(\varepsilon^2)\right). && (45) \end{aligned}$$

These results confirm that B/R^2 is sufficiently small to justify that we neglected some terms in Eq. (37), hence, we have found consistent solutions for R and B. More support for the validity of the approximations is provided by the numerical solution of the complete Eq. (37) in Fig. 1.

With R and B as functions of only the matter-related term r, it is possible to calculate $L_{1,2}$ in (25). First, we have to define the square root of x^2 in (20). Since we work in the limit $|R^2/B| \gg 1$, which implies $|f_Q R| = |mzR/B| \gg z/R$, we identify $f_Q R$ to be the dominant term in Eqs. (30) and (18). Thus $f_R \approx -2 f_Q R$ and $x^2 \approx \frac{1}{4} f_R^2 \approx (f_Q R)^2$, and we choose the convention $x = f_Q R \sqrt{1 + \cdots}$, where $\cdots$ denotes the remaining terms in Eq. (18) divided by $(f_Q R)^2$. Next, we plug $x \approx -\frac{1}{2} f_R \approx f_Q R$ into the trace equation (22)

$$c_a(2 f_Q R + 2 f_R) = 3x + c_b \cdot x \sqrt{1 - (2 f_Q r) x^{-2}}, \tag{46}$$

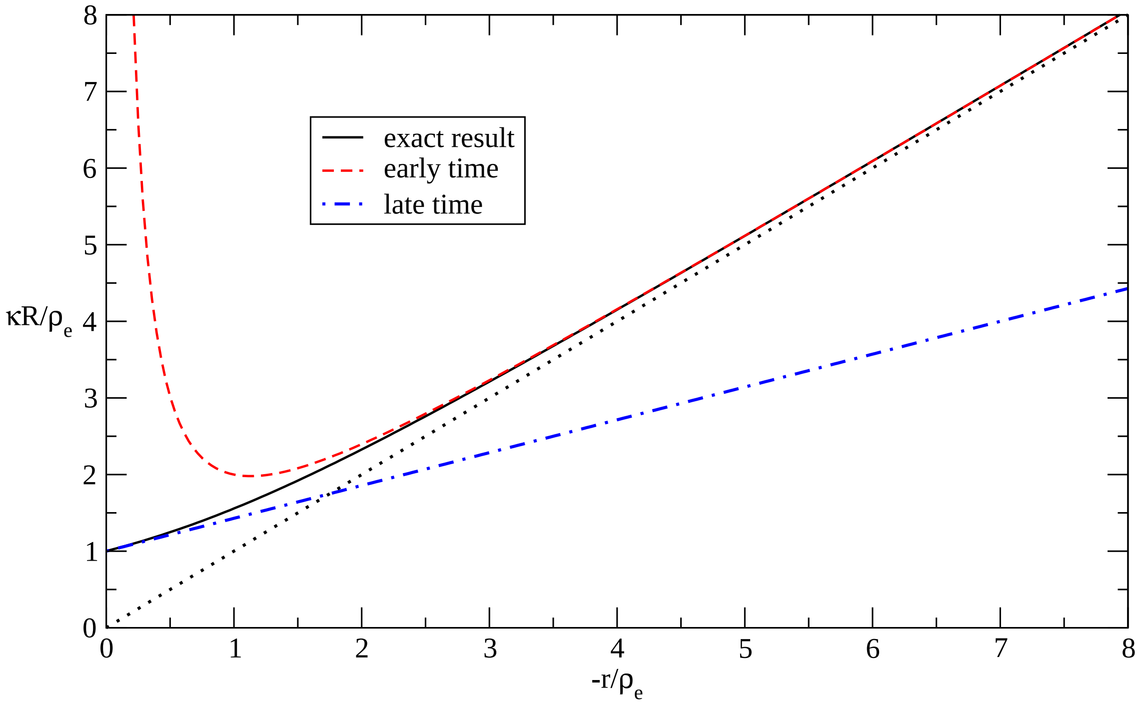

Figure 1: Numerical solutions for $\kappa R < 0$ as a function of $r = \rho$ with $p = 0$. The plot shows the exact result in Eq. (37) (black solid curve) and the approximations at early times (red dashed) from Eq. (40) as well as at late times (blue dashed-dotted) from Eq. (41), respectively. In this example with $m = 3$ the parameter $\beta < 0$ has been chosen such that $-|\Lambda| \approx 358\,\rho_e$ only for numerical reasons. A more realistic ratio of $|\Lambda/\rho_e|$ would be much larger, however the results would not differ qualitatively, which is true also for other values of the EOS p/ρ as long as $r \ll |\Lambda|$. In any case, κR approaches $-r$ (dotted diagonal line) in the region $-\rho_e \ll r \ll |\Lambda|$.

which gives in leading order $c_a(-2)f_Q R = f_Q R(3+c_b)$. Obviously, $c_a = c_b = -1$ is the only solution[4] in the large $|f_Q R|$ limit, and we find

$$\begin{aligned}
L_1 &= -x + \frac{1}{2} f_R \\
&= -2f_Q R + R^{-1}\left(mz + \frac{3}{4}\kappa R - \frac{1}{4}r\right) + \frac{(mz)^2}{6f_Q R^3}\left(1+\varepsilon\right), \qquad (47)\\
L_2 &= -x - x\sqrt{1-(2f_Q r)x^{-2}} \\
&= -2f_Q R + R^{-1}\left(\frac{4}{3}mz + \frac{1}{2}(\kappa R + r)\right) + \frac{(mz)^2}{3f_Q R^3}\left(1+\varepsilon\right), \qquad (48)
\end{aligned}$$

where we used a series expansion of the root in

$$\begin{aligned}
f_R &= -2f_Q R + R^{-1}\left(\kappa R + \frac{2}{3}mz\right), \qquad (49)\\
x &= f_Q R\sqrt{1 - \frac{8mz + 3\kappa R - 3r}{6f_Q R^2} + \frac{(2mz+3\kappa R)^2}{36 f_Q^2 R^4}}. \qquad (50)
\end{aligned}$$

Above and from here on ε denotes small terms of the order $\kappa R/z$, r/z or B/R^2. Moreover, Λ has been eliminated in favour of

$$z = -\frac{3}{6+4m}\left(\kappa R - 4\Lambda + 3p - \rho\right). \qquad (51)$$

Finally, we write down the series expansion of

$$L_1 - L_2 = -\frac{1}{3}\frac{mz}{R}\left(1 + \frac{1}{2}\frac{B}{R^2}\left(1+\varepsilon\right)\right), \qquad (52)$$

which appears in the auxiliary metric h_{ab}. Now, we have all ingredients available for discussing solutions to the Palatini field equations in the next sections.

4 Cosmology

For investigating the cosmological evolution the physical metric is of the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) type with $g_{00} = -1$ and $g_{ii} = a^2(t)$ in Cartesian coordinates, where $a(t)$ is the scale factor as a function of cosmological time t. The matter component is at rest ($u^m = -\delta_0^m$) in these coordinates. Accordingly, the auxiliary metric h_{ab} in Eq. (26) is completely defined by the diagonal elements $h_{ii} = \Omega g_{ii}$ and $h_{00} = \Omega(g_{00} - L_2/(L_1 - L_2))$, which follow explicitly from Eqs. (47), (48) and (52),

$$\begin{aligned}
\Omega &= \frac{\sqrt{|L_1(L_1-L_2)|}}{L_1} = \mathrm{sgn}(L_1)\sqrt{|\frac{2}{3}(mz)^2 B^{-1}|}\left(1+\varepsilon^2\right), \qquad (53)\\
h_{00} &= \Omega\frac{-L_1}{L_1 - L_2} = \Omega\left(-6\frac{R^2}{B}\right)\left(1 - \frac{B}{R^2}\left(1+\varepsilon\right)\right). \qquad (54)
\end{aligned}$$

[4]Note that our scenario with $c_b = -1$ relies from the very beginning on the existence of matter, because otherwise the vector u_m in Eq. (16) were absent and $c_b = +1$ would be required in Eq. (21). However, matter is a fact of reality, which singles out $c_b = -1$ in our model. Consequently, Eq. (41) is the correct solution in the $r \to 0$ limit.

Note that if B and L_1 are positive, the metric h_{ab} has the same signature as g_{ab}. Moreover, in the following z will be treated as a time-independent constant proportional to the CC Λ since the corrections can be subsumed in the $\varepsilon \sim \mathcal{O}(\kappa R/z)$ terms. At this point we are ready to apply the results for B and R^2/B that we found in Eqs. (42), (43), (44) and (45), respectively.

4.1 Early universe

We begin with the epoch, where the matter energy density ρ (including dust and radiation) is well below the large CC $\Lambda \sim z$ in magnitude but above the asymptotic future energy density $|\rho_e|$. Therefore, our discussion will be valid for most parts of the radiation and matter eras. According to (53) and (54) we find

$$\Omega = c_1 \left(\frac{D}{L_3}\right)^{\frac{1}{4}} \left(1 + e\, e_1 \frac{D}{3r} + \mathcal{O}(e^2)\right), \quad c_1 = \text{const.} \tag{55}$$

$$h_{00} = c_1 \left(-\frac{4}{3} mz/e\right) L_3^{-\frac{1}{4}} D^{-\frac{3}{4}} \left(1 - e\, e_2 \frac{D}{r} + \mathcal{O}(e^2)\right), \tag{56}$$

where $r = \rho + p$ and $D = (L_3/r^4)^{(1/3)}$ was introduced earlier in Eq. (40). The sign under the root in the constant $c_1 = \sqrt{\pm 3(\frac{2}{9}mz)^3(-\kappa)^2}$ may be chosen such that consistency with Eq. (53) is obtained. However, since c_1 drops out from the connection $\Gamma(h)$ in Eq. (12) we skip this question for the moment. In addition, we have introduced above the purely technical parameter $e = 1$ which counts the powers of the small quantity $|D| \ll r$, whereas $e_{1,2} = 1$ just denote first-order correction terms.

According to Eq. (12) the non-zero components of the (symmetric) connection read (with $i = 1, 2, 3$)

$$\Gamma^0_{00} = \frac{1}{2}\frac{\dot{r}}{r} + e\, e_2 \frac{7}{6}\frac{\dot{r}}{r}\frac{D}{r} + \mathcal{O}(e^2), \tag{57}$$

$$\Gamma^0_{ii} = e \frac{a^2}{8mz} D \left(6\frac{\dot{a}}{a} - \frac{\dot{r}}{r}\right) + \mathcal{O}(e^2), \tag{58}$$

$$\Gamma^i_{i0} = \frac{1}{6}\left(6\frac{\dot{a}}{a} - \frac{\dot{r}}{r}\right) - e\, e_1 \frac{7}{18}\frac{\dot{r}}{r}\frac{D}{r} + \mathcal{O}(e^2). \tag{59}$$

For a simpler presentation we assume from now on a constant matter EOS $\omega = p/\rho > -1$, which yields a simple expansion law for our matter component via its conservation equation[5]

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = 0 \quad \Leftrightarrow \quad \rho \propto a^{-3(1+\omega)}. \tag{60}$$

Hence, from Eq. (4) we obtain for the non-zero components $R_0 := R_0^{\ 0}$ and $R_i := R_1^{\ 1} = R_2^{\ 2} = R_3^{\ 3}$ of the diagonal Ricci tensor $R_a^{\ b} = g^{cb}R_{ac}$ the following results,

$$\begin{aligned} R_0 &= g^{00}R_{00} = \frac{3}{2}(\omega+3)\left(2(\omega+1)\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a}\right) \\ &+ e\frac{7}{4}\frac{D}{\rho_m}\left[\left(\frac{\dot{a}}{a}\right)^2 (e_1(19\omega+21) + e_2 3(\omega+3)) + e_2 2\frac{\ddot{a}}{a}\right] + \mathcal{O}(e^2), \end{aligned} \tag{61}$$

[5] Since the matter action $\mathcal{S}_{\rm mat}$ in (3) does not involve the Palatini connection, the covariant derivative $\nabla_n = \nabla_n[g]$ in the matter conservation equation $\nabla_n T_m^{\ n} = 0$ contains only the Christoffel symbols of the physical metric g_{ab}.

$$R_i \;=\; g^{11}R_{11} = e\frac{3D}{8mz}(\omega+3)\left[\left(\frac{\dot a}{a}\right)^2(3\omega+5)+\frac{\ddot a}{a}\right]+\mathcal{O}(e^2). \tag{62}$$

Remember that the Palatini connection Γ^a_{bc} given above and all derived quantities like R or R_{ab} are different from the metric versions, and they should not be compared with them. Observables are related to the scale factor a in the physical metric g_{ab}. Due to $R_i = \mathcal{O}(e)$ the leading component of the Ricci scalar $R = R_0 + 3R_i$ is just the $\mathcal{O}(e^0)$-term in R_0:

$$R = \frac{3}{2}(\omega+3)\left(2(\omega+1)\left(\frac{\dot a}{a}\right)^2+\frac{\ddot a}{a}\right)+\mathcal{O}(e). \tag{63}$$

Comparing this result with $\kappa R = -\rho(1+\omega)+\mathcal{O}(e)$ from Eq. (40), we quickly deduce the leading scale factor behaviour. The situation is similar to general relativity because the power-law ansatz $a(t)\propto t^s$, $s=$ const. for the scale factor represents a reasonable solution. Equipped with the corresponding Hubble rate $H = s/t$ and $\ddot a/a = s(s-1)/t^2$, Eq. (63) yields $R\propto t^{-2}$ at zero order, and $\rho \propto a^{-3(1+\omega)} \propto t^{-3s(1+\omega)}$ will be proportional to R if

$$s = \frac{2}{3(\omega+1)} \quad \text{in} \quad a(t)\propto t^s. \tag{64}$$

Hence, we find from Eq. (63)

$$H = \frac{2}{3(\omega+1)t},\quad \frac{\ddot a}{a} = H^2\frac{1+3\omega}{2},\quad R = \frac{3}{4}(\omega+3)^2H^2+\mathcal{O}(e) = (-\kappa)^{-1}\rho(1+\omega), \tag{65}$$

implying the modified Friedmann equation

$$(-\kappa)H^2 = \frac{4}{3}\frac{\omega+1}{(\omega+3)^2}\,\rho, \tag{66}$$

which is the main result of this section. Accordingly, a dust dominated universe with EOS $\omega = 0$ leads to

$$(-\kappa)H^2 = \frac{4}{27}\rho, \quad \text{(dust)} \tag{67}$$

which is not very different from the radiation dominated cosmos with $\omega = \frac{1}{3}$. In the latter case, the modified Friedmann equation reads

$$(-\kappa)H^2 = \frac{4}{25}\rho, \quad \text{(radiation)} \tag{68}$$

where ρ is the radiation energy density. Obviously, dust matter and radiation influence the cosmic expansion in almost the same way as in standard general relativity if we choose the parameter $\kappa = -\frac{4}{27}\cdot 3/(8\pi G_N)$ with Newton's constant G_N. However, comparing Eq. (67) with (68) indicates an increased expansion rate in the radiation era, which can be expressed by a higher effective Newton constant $G_{\text{rad}} = \frac{27}{25}G_N = 1{,}08\,G_N$. This 8% difference seems to be well within current bounds on the variation of G_N. For instance, in the context of Big Bang nucleosynthesis the bound $G_{\text{rad}}/G_N = 1{,}10\pm 0{,}07$ was given recently in Ref. [57], which can be related also to constraints on (additional) relativistic degrees of freedom [58, 59, 60]. Accordingly, the small difference in the expansion rates above is not in conflict with recent observations.

As a more technical point we remark that $\kappa = +1/(8\pi G_N)$ is positive in general relativity, however, in our setup the negative term $\kappa R < 0$ is only one part in the action (29),

which is completed by the crucial term z. Consequently, $\kappa < 0$ does not correspond to a negative Newton constant in general relativity. Another interesting feature of Eq. (66) is that even if we had allowed additional vacuum energy ($\omega = -1$) contributions in ρ, it would not influence the expansion rate H because of the vanishing right-hand side of Eq. (66). An example for this are finite shifts $|\delta\Lambda| \ll |\Lambda|$ in the vacuum energy density emerging from phase transitions.

Comparing the Palatini setup with the CC relaxation models in the LXCDM framework [14, 15] and in metric $f(R,G)$ [31, 1] modified gravity, we find another advantage. Here, it is not necessary to include a parameter which controls the transition from radiation to (dust) matter domination. It simply happens when the energy density $\rho_{\text{dust}} \propto a^{-3}$ of dust matter overtakes the radiation density $\rho_{\text{radiation}} \propto a^{-4}$ as a result of the growing scale factor a. Therefore, the relation of the energy content with the expansion of the universe is very similar to general relativity with a negligible CC.

For completeness we list all results including a first order correction in the scale factor

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(\omega+1)}} \left(1 + e_a\, e \frac{D}{\rho(1+\omega)}\right), \tag{69}$$

with the constant e_a to be determined below. The components of the diagonal Ricci tensor read

$$\begin{aligned} R_0 &= \frac{(\omega+3)^2}{3t^2(1+\omega)^2}\left(1 + e\frac{D}{\rho}\left[\frac{7[4e_1(5+4\omega) + 3(3+\omega)(e_2 + e_a(23+19\omega))]}{3(3+\omega)^2}\right] + \mathcal{O}(e^2)\right), \\ R_i &= e\frac{D(\omega+3)^2}{4mzt^2(1+\omega)^2} + \mathcal{O}(e^2), \end{aligned} \tag{70}$$

yielding the Ricci scalar $R = R_0 + 3R_i$ and the squared Ricci tensor Q,

$$R = R_0 + \frac{(\omega+3)^2}{3t^2(1+\omega)^2}\left(1 + e\frac{9}{4}\frac{D}{mz} + \mathcal{O}(e^2)\right) \tag{71}$$

$$Q = R_a{}^b R_b{}^a = (R_0)^2 + 3(R_i)^2 = (R_0)^2 + \mathcal{O}(e^2). \tag{72}$$

Thus, the function B in the denominator of z in Eq. (29) is given by

$$B = R^2 - Q = 6R \cdot R_i - 12(R_i)^2 = e\frac{D(3+\omega)^4}{mz(2t^4)(1+\omega)^4} + \mathcal{O}(e^2), \tag{73}$$

which evidently proves that $B/R^2 = \mathcal{O}(D/z)$ is a small quantity justifying a posteriori our series expansions. In addition, since B is relatively small but finite, the term z in the action (29) does not diverge.

Finally, let us determine the coefficient e_a in the scale factor correction term by comparing R in (71) with Eq. (40), yielding $e_a = -\frac{10}{63}$ for dust ($\omega = 0$) and $e_a = -\frac{817}{6160}$ for radiation ($\omega = \frac{1}{3}$), respectively. Note that the term $e\frac{9}{4}\frac{D}{mz}$ in R does not come from the first-order corrections $\propto e_{1,2,a}$ in Eqs. (55) and (56), but it emerges as the leading term of R_i and it provides the correct (non-zero) value for B. With these results one can check explicitely that $z = \beta(R^{(2/3)}/B)^m$ and the validity of the EOMs in Eqs. (7) and (8).

4.2 Late universe

Analogously to the discussion on the early universe in the previous section, we determine now the scale factor evolution at late times, when $r = \rho + p \ll |\rho_e|$ and $\rho_e < 0$. After

plugging B and B/R^2 from Eqs. (44) and (45) into the expressions (53) and (54) for the auxiliary metric h_{ab}, we find

$$\begin{aligned}
\Omega &= c_2 \rho_e^{-\frac{3}{2}} \left(1 + e\, e_1 \frac{1}{7} \frac{r}{\rho_e} + \mathcal{O}(e^2)\right), && (74) \\
h_{00} &= c_2 \left(-\frac{4}{3} mz/e\right) \rho_e^{-\frac{5}{2}} \left(1 - e\, e_2 \frac{3}{7} \frac{r}{\rho_e} + \mathcal{O}(e^2)\right). && (75)
\end{aligned}$$

Also here, the minus sign in ρ_e should be absorbed in the constant $c_2 = \sqrt{\pm \frac{27}{2} (\frac{2}{9} mz)^3 (-\kappa)^2}$, which drops out when calculating the connection $\Gamma(h)$ in (12). Like before $e = 1$ represents powers of suppressed terms like $B/R^2 = \mathcal{O}(\rho_e/z)$ or r/ρ_e, and $e_{1,2} = 1$ signal first-order corrections. The non-vanishing components of the connection are given by (with $i = 1, 2, 3$)

$$\begin{aligned}
\Gamma^0_{00} &= -e\, e_2 \frac{3}{14} \frac{\dot{r}}{\rho_e} + \mathcal{O}(e^2), && (76) \\
\Gamma^0_{ii} &= e \frac{3a^2}{4mz} \rho_e \frac{\dot{a}}{a} + \mathcal{O}(e^2), && (77) \\
\Gamma^i_{i0} &= \frac{\dot{a}}{a} + e\, e_1 \frac{1}{14} \frac{\dot{r}}{\rho_e} + \mathcal{O}(e^2). && (78)
\end{aligned}$$

Furthermore, we assume a constant EOS ω for the matter content as in Eq. (60), which simplifies the expressions for the Ricci tensor components $R_0 = R_0{}^0$ and $R_i = R_1{}^1$ from Eq. (4),

$$\begin{aligned}
R_0 &= 3\frac{\ddot{a}}{a} + e \frac{9}{14} (1+\omega) \frac{r}{\rho_e} \left((e_1(2+3\omega) - 3e_2) \left(\frac{\dot{a}}{a}\right)^2 - e_1 \frac{\ddot{a}}{a} \right) + \mathcal{O}(e^2), \\
R_i &= e \frac{3\rho_e}{4mz} \left(2 \left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} \right) + \mathcal{O}(e^2). && (79)
\end{aligned}$$

As before, the scale factor behaviour can be found by comparing the zero-order term in the Ricci scalar $R = R_0 + 3R_i = 3\frac{\ddot{a}}{a} + \mathcal{O}(e)$ with Eq. (41) in the limit $r \to 0$,

$$\kappa\, 3 \frac{\ddot{a}}{a} = \rho_e. \tag{80}$$

The solution is a linear combination of exponential functions, $a(t) \propto \exp(\pm H_e t)$ with constant H_e. Since the shrinking solution quickly decays at late times we drop it and consider only the de Sitter-like solution

$$a(t) \propto \exp(H_e t) \left(1 + e\, e_a \frac{r}{\rho_e}\right), \tag{81}$$

where a first-order correction proportional to the constant e_a has been added. Consequently,

$$\begin{aligned}
&H = H_e \left(1 + e\, e_a \frac{-3(1+\omega) r}{\rho_e} + \mathcal{O}(e^2)\right), \quad \frac{\ddot{a}}{a} = H_e^2 \left(1 + e\, e_a \frac{3(1+3\omega)(1+\omega) r}{\rho_e} + \mathcal{O}(e^2)\right), \\
&R_0 = 3H_e^2 \left(1 + e \frac{3(3\omega - 2 + 14 e_a (1+3\omega))(1+\omega) r}{14 \rho_e} + \mathcal{O}(e^2)\right), \quad R_i = e \frac{9H_e^2 \rho_e}{4mz} + \mathcal{O}(e^2),
\end{aligned} \tag{82}$$

where $e_{1,2} = 1$ was applied. Neglecting the tiny $\mathcal{O}(\rho_e/z)$-term in R_i, the Ricci scalar $R = R_0 + 3R_i$ will be consistent with Eq. (41), $\kappa R = \rho_e - \frac{3}{7}r$, if

$$\rho_e = 3\kappa H_e^2 \;\text{ and }\; e_a = \frac{-\omega}{14(1+\omega)}. \tag{83}$$

From the first condition the parameter β in $L_3 = \rho_e^7 < 0$ can be determined because $-\kappa \sim 1/G_N$ was suggested in Sec. 4.1 and thus $-\rho_e \sim -\kappa H_e^2$ must be of the order of the late-time critical energy density. This is just the effective CC corresponding to the de Sitter solution (81) with the tiny observed Hubble rate H_e. For dust matter ($\omega = 0$) the scale factor correction $\propto e_a$ vanishes and the expansion is purely de Sitter-like.

As in the previous section, the squared Ricci tensor reads $Q = (R_0)^2 + \mathcal{O}(e^2)$ and we find that the denominator of z in Eq. (29) is highly suppressed but non-zero,

$$B = R^2 - Q = 6R \cdot R_i - 12(R_i)^2 = e\frac{81H_e^4\rho_e}{2mz} + \mathcal{O}(e^2). \tag{84}$$

Therefore, our expansion in $B/R^2 = \mathcal{O}(\rho_e/z)$ turns out to be justified also at late times.

To finalise this section we estimate the magnitude of the parameter β expressed as a power of an energy scale M_β. The dimensional analysis of z in Eq. (29) implies $|\beta| = M_\beta^d$ with the exponent given by $d = 4 + \frac{8}{3}m$. From Eq. (32) we know that z is of the order of the large cosmological term Λ, and in Eq. (83) we related $(-\rho_e) \sim (10^{-12}\,\text{GeV})^4$ to the tiny observed energy density of the effective late-time CC. Finally, in Sec. 4.1 the parameter $(-\kappa) \sim (10^{18}\,\text{GeV})^2$ was fixed by the inverse Newton constant, and now we are prepared to estimate β for given values of Λ and m by using $\rho_e^7 = L_3$ in Eq. (39),

$$\beta = \left(\frac{\rho_e^7}{\kappa^4}\right)^{\frac{m}{3}} \left(\frac{9}{2m}\right)^m z^{(1-m)}. \tag{85}$$

First, we note that with large values of m the tiny first factor in β produces small values of $M_\beta = |\beta|^{1/d}$, which makes this parameter range less attractive. On the other hand, $m < 1$ provides an interesting range of energy scales. For instance, for the vacuum energy density $|\Lambda| \sim |z| \sim (10^{16}\,\text{GeV})^4$ of a typical grand unified theory we obtain the following magnitudes of M_β in units of GeV: 10^{-51} for $m = 3$, 10^{-2} ($m = \frac{1}{3}$), 10^4 ($m = \frac{1}{5}$), 10^{12} ($m = \frac{1}{17}$), and M_β approaches z in the limit $m \to 0$. Apart from that, the structure of Eq. (85) does not require β to be fixed very precisely, which constitutes a much better situation compared to the counterterm method in Eq. (1).

5 Kottler solution

In the previous section we have seen that the universe approaches a de Sitter cosmos in the limit of vanishing matter, where $r \to 0$ and $\kappa R \to \rho_e$ in Eq. (41). We use this result now for discussing the Kottler (Schwarzschild-de Sitter) solution, which describes a Schwarzschild black hole in the presence of a positive CC. For analysing the CC filter in this environment we need the 4-velocity vector u_m from Eq. (26), which we know already in the cosmological setup. Therefore, it is useful to apply a transformation from the spatially flat FLRW coordinates (t, ρ, θ, ϕ) with line element

$$ds^2 = -dt^2 + a^2(t)\left(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2\right) \tag{86}$$

into Lemaître coordinates $(\tau, \sigma, \theta, \phi)$, where the angles θ and ϕ remain untouched,

$$ds^2 = -A\, d\tau^2 + A^{-1} d\sigma^2 + \sigma^2 d\theta^2 + \sigma^2 \sin^2\theta d\phi^2. \tag{87}$$

In the latter metric the mass of the black hole in terms of the Schwarzschild radius r_s and respectively the de Sitter radius r_e define the term

$$A(\sigma) = 1 - \frac{r_s}{\sigma} - \frac{\sigma^2}{r_e^2}. \tag{88}$$

The radial coordinates are related by $\sigma = a(t)\rho$ and if we require that $A = 1 - (H\sigma)^2$ with the Hubble rate $H = \frac{\dot{a}}{a}$, the transformation rules between both metrics in (86) and (87) read

$$\frac{\partial\sigma}{\partial t} = H\sigma = \sqrt{1-A},\ \frac{\partial\sigma}{\partial\rho} = a,\ \frac{\partial\tau}{\partial t} = \frac{1}{A},\ \frac{\partial\tau}{\partial\rho} = \frac{aH\sigma}{A} = \frac{a\sqrt{1-A}}{A}. \tag{89}$$

Next, we consider the 4-velocity vector u_m in (26), which has only one non-vanishing component $u_t = 1$ in FLRW coordinates. Via the relations (89) the corresponding components in Lemaître coordinates can be obtained easily,

$$u_\tau = 1,\ u_\sigma = -\frac{H\sigma}{A} = -\frac{\sqrt{1-A}}{A}. \tag{90}$$

Of course, the norm $u_m u^m = -1$ remains invariant under this change. With Eqs. (87) and (90) the auxiliary metric in (26) is given by

$$\begin{aligned}
h_{\tau\tau} &= \Omega\,(g_{\tau\tau} - L_u u_\tau u_\tau) = \Omega\,(-A - L_u) \\
h_{\sigma\sigma} &= \Omega\,(g_{\sigma\sigma} - L_u u_\sigma u_\sigma) = \Omega\left(A^{-1} - L_u\frac{1-A}{A^2}\right), \\
h_{\tau\sigma} &= \Omega\,(g_{\tau\sigma} - L_u u_\tau u_\sigma) = \Omega\left(0 + L_u\frac{\sqrt{1-A}}{A}\right), \\
h_{mn} &= \Omega\, g_{mn} \ \text{ for the other components.}
\end{aligned} \tag{91}$$

Interestingly, h_{mn} has non-zero off-diagonal elements, whereas the physical metric g_{mn} does not. Here, we have introduced the variable L_u, which follows from Eqs. (48), (52) and (45), and its leading term reads

$$L_u := \frac{L_2}{L_1 - L_2} = 6\frac{R^2}{B}(1+\varepsilon) = 6\,\frac{\frac{2}{9}mz}{\rho_e}(1+\varepsilon). \tag{92}$$

Since we consider the limit $r \to 0$, both terms Ω and L_u are constant. Thus Ω will drop out from the connection Γ^a_{bc} according to Eq. (12). For the non-vanishing components we find

$$\begin{aligned}
\Gamma^\tau_{\tau\tau} &= -\frac{L_u}{1+L_u}\cdot\frac{\sqrt{1-A}}{2\sigma A}\left(\frac{r_s}{\sigma} + 2\frac{\sigma^2}{r_e^2}\right) = A^2(1-A)\Gamma^\tau_{\sigma\sigma} = -\Gamma^\sigma_{\tau\sigma}, \\
\Gamma^\tau_{\tau\sigma} &= -\frac{A + L_u(A-1)}{(1+L_u)2\sigma A}\left(\frac{r_s}{\sigma} + 2\frac{\sigma^2}{r_e^2}\right),\ \Gamma^\tau_{\theta\theta} = -\frac{L_u\sigma\sqrt{1-A}}{(1+L_u)A} = \sin^{-2}\theta\,\Gamma^\tau_{\phi\phi} \\
\Gamma^\sigma_{\tau\tau} &= -\frac{A+L_u}{(1+L_u)2\sigma}\left(\frac{r_s}{\sigma} + 2\frac{\sigma^2}{r_e^2}\right),\ \Gamma^\sigma_{\sigma\sigma} = -\Gamma^\tau_{\tau\sigma},\ \Gamma^\sigma_{\theta\theta} = -\frac{\sigma(A+L_u)}{1+L_u} = \sin^{-2}\theta\,\Gamma^\sigma_{\phi\phi} \\
\Gamma^\theta_{\sigma\theta} &= \frac{1}{\sigma} = \Gamma^\phi_{\sigma\phi},\ \Gamma^\theta_{\phi\phi} = -\cos\theta\sin\theta,\ \Gamma^\phi_{\phi\theta} = \cot\theta.
\end{aligned} \tag{93}$$

Moreover, the non-zero components of the Ricci tensor read

$$\begin{aligned} R_{\tau\tau} &= -\frac{3(A+L_u)}{r_e^2(1+L_u)}, \;\; R_{\tau\sigma} = R_{\sigma\tau} = \frac{3L_u\sqrt{1-A}}{r_e^2(1+L_u)A}, \\ R_{\sigma\sigma} &= -\frac{3(L_u(1-A)-A)}{r_e^2(1+L_u)A^2}, \;\; R_{\theta\theta} = \frac{3\sigma^2}{r_e^2(1+L_u)} = \sin^{-2}\theta\, R_{\phi\phi}. \end{aligned} \tag{94}$$

Also here we find non-diagonal entries, however, they belong to the Palatini Ricci tensor, which is derived from h_{mn} and not from the physical metric. Finally, we show the results for the scalar invariants

$$R = R_{ab}g^{ab} = \frac{3}{r_e^2}\cdot\frac{4+L_u}{1+L_u}, \tag{95}$$

$$\begin{aligned} Q &= R_{ab}R_{cd}g^{ac}g^{bd} = \frac{(R_{\tau\tau})^2}{A^2} + 2(R_{01})^2\frac{A}{-A} + A^2(R_{\sigma\sigma})^2 + \frac{(R_{\theta\theta})^2}{\sigma^4} + \frac{(R_{\phi\phi})^2}{\sin^4\theta\sigma^4} \\ &= \left(\frac{3}{r_e^2}\right)^2\cdot\frac{4+2L_u+L_u^2}{(1+L_u)^2}, \end{aligned} \tag{96}$$

$$B = R^2 - Q = \left(\frac{3}{r_e^2}\right)^2\cdot\frac{6(2+L_u)}{(1+L_u)^2} = R^2\frac{6}{L_u}(1+\mathcal{O}(\varepsilon)). \tag{97}$$

At leading order the last equation is consistent with Eq. (45), and it implies that $B/R^2 = \mathcal{O}(\rho_e/z) \ll 1$ is a suitable expansion parameter. Moreover, by comparing R in Eq. (95) with the late-time results from Eqs. (41) and (83), we find at leading order

$$R = \frac{3}{r_e^2} = \frac{\rho_e}{\kappa} = 3H_e^2. \tag{98}$$

This implies that the de Sitter radius r_e as a parameter in the Kottler metric (87) is given by the inverse of the final Hubble rate $H_e = r_e^{-1}$, just as in general relativity. Remember that H_e originates from the small effective vacuum energy density of the order $|\rho_e|$ and not from the large CC Λ. Note also that Eqs. (90) and (98) are sufficient to show that the metric (87) with A given in (88) is a solution of our Palatini model. The coordinate transformation (89) just served us to obtain the vector u_m in Lemaître coordinates. As a result of this section, the CC is relaxed also in situations, which can be well described by the Kottler metric. This can be very useful for solar system tests and the construction of vacuole solutions of the Einstein-Strauß type [61, 62], which we would like to discuss in the future. For the vacuole solutions we cannot assume $r \ll |\rho_e|$ as we did in Sec. 4.2, but the matter density r must be treated on equal footing with the effective vacuum energy density $|\rho_e|$.

6 Conclusions and outlook

In the context of the old CC problem, we have presented a modified gravity model, which filters out a large CC Λ independent of its origin. Thanks to the Palatini formalism, we avoid problems coming from extra degrees of freedom, which are often present in the metric formalism. In this work, several aspects of our filter scenario have been analysed with the result that the standard Big Bang history of the universe is not prevented by a large CC term even when it dominates in size over matter or radiation.

Unlike many models for cosmological late-time acceleration, our setup in Eq. (29) does not represent a small correction to the Einstein-Hilbert term because the term z containing the Ricci scalar R and the squared Ricci tensor Q, plays a crucial role for the filter effect. Thus, it is neither useful nor necessary to consider the limit $z \to 0$. Furthermore, we have shown that the Hubble expansion rate in the early universe is dominated by the (non-vacuum) matter energy density, whereas CC contributions with equation of state $\omega = -1$ do not contribute at all in leading order. This filter effect implies a cosmological background evolution similar to general relativity, where the large CC has been removed somehow. We have found reasonable results in the matter and radiation eras. In the latter the effective Newton constant is slightly increased but consistent with recent bounds [57]. At late times, when matter has diluted away, we enter a de Sitter phase, where the expansion rate H_e is not dominated by the large CC, but instead it depends only on the magnitudes of the parameter β and Λ in Eq. (39). Hence, the inclusion of a CC counterterm which cancels Λ with extremely high precision is not necessary for describing the currently observed accelerated expansion. Our cosmological results are supported by the existence of a black hole solution of the Kottler type, in which the effective CC parameter complies with the value of the final de Sitter expansion rate H_e. This suggests that cosmology and the astro-physical domain can be smoothly connected.

The robustness of the CC filter can be seen also from a different perspective. So far we have considered Λ to be a large constant generally of the order of the largest contribution to the CC. During the cosmic evolution, however, it is not unlikely that shifts $\delta\Lambda$ of vacuum energy occur, as a result of phase transitions for instance. We have shown that these contributions, which are smaller than Λ by definition, have no influence on gravity at leading order if we treat them as part of the matter sector described by $r = \rho + p$ in Eq. (37). Alternatively, one could shift the cosmological term by $\Lambda \to \Lambda + \delta\Lambda$, but also this procedure does not change our results because the differences are of the order $|\delta\Lambda/\Lambda| \ll 1$ contributing only to the small correction terms. Hence, the exact value of Λ is not important for the CC filter effect even when vacuum shifts $\delta\Lambda$ arise dynamically as the universe evolves. Note that $\delta\Lambda$ can be much larger than the current critical energy density despite being small compared to Λ.

Let us now sketch some open points and discuss possible generalisations of our setup, which we want to address in the future. First, it is important to mention that our filter effect is based on the largeness of the CC and its universal energy-momentum structure. This can be seen nicely in Eq. (34), where the CC contribution to the Ricci scalar R can be completely removed by a suitable model building construction, which leads to Eq. (37). It is clear that as long as Λ dominates over other sources in the energy-momentum tensor and its trace (31), the procedure will be the same, which removes from R the dominating term $z \sim \Lambda$ as given in Eq. (32). Therefore, we can at least conjecture that matter sources different from the perfect fluid form as well as other background metrics exhibit also the CC filter property. This would be useful for future studies of the Newtonian limit as well as perturbations in cosmology and astro-physical setups, which are more involved than the Kottler solution. For the latter, it seems that vacuole space-times can be easily constructed once a solution is found which interpolates between the matter era and the final de Sitter phase. The smoothness of R as shown in Fig. 1 suggests such a solution. More difficult could be the analysis of the very early cosmological epoch when matter dominated over Λ (if there was such a time) or during primordial inflation, respectively. In that case many approximations we used for the subsequent eras cannot be applied anymore, and a new analysis is necessary.

Apart from that, we have shown only one numerical example to support our analytical considerations, and it would be nice to have more numerical results discussing e.g. the transition phases between different epochs and the constraints on the parameters β and m. Also the $L_3 > 0$ case in Eq. (39) might correspond to an interesting solution, maybe it describes a contracting universe at late times because a vanishing matter density is not a solution of Eq. (39). However, since we observe an accelerating cosmos, the $L_3 < 0$ case discussed here is preferred.

Moreover, we should mention that Palatini models are subject to constraints from astrophysical bodies and even small scale atomic physics, see e.g. [63, 35, 47, 48, 37]. However, many results correspond to $f(R)$-type actions or to models, where the Einstein-Hilbert term is amended by a correction, which becomes large for low curvature. It is clear that our setup is not of this type. We will see in the future whether this is an advantage or not. Nevertheless, it might help that the Ricci scalar in Eq. (41) and the denominator function B in (44) approach a non-zero constant in the limit of vanishing matter. This suggests a stable vacuum, which limits the values of the geometrical scalars from below. Finally, it has been argued in Refs. [47, 48] that the averaging over matter sources in the Palatini framework has some non-trivial aspects, too.

In the end, it would be interesting to know whether the CC filter effect is just a curiosity of our specific model in Eq. (29), or if there are completely different choices, which have the same property. We want address these questions in the future.

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